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The Abelian Higgs Model in Three Dimensions with Improved Action

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ABSTRACT

We study the Abelian Higgs Model using an improved form of the action in the scalar sector. The subleading corrections are carefully analysed and the connection between lattice and continuous parameters is worked out. The simulation shows a remarkable improvement of the numerical performance.

1 Introduction

The Abelian Higgs Model has mostly been studied in recent years as a theoretical laboratory in the context of the Electroweak Baryogenesis scenario. As it is well known by now, the lattice investigations of the model are very demanding in computer power and time. It would be helpful to use an improved form of the lattice action, to reduce the autocorrelation and come closer to the continuum limit.

We did not try an improvement on the pure gauge sector of the action. The strategy which is usually followed does not seem very reliable: actually it leaves behind several subleading contributions, so it is not very efficient in eliminating all of the unwanted terms. On the other hand, the weak coupling regime where these simulations are performed suggest that the improvement of the gauge sector may not be very important. We mainly concentrated on improving the scalar sector; the procedure suffers more or less from the same problems, however even a modest improvement is very important in this case. The model on which we will work has been treated before ([1]), so we have reference results to compare with.

A promising approach to study the four-dimensional model at finite temperature is through reduction to an effective model in three dimensions. This can be done if the couplings are small and the temperature is much larger than any other mass scale in the theory. The parameters of the reduced theory are related to the ones of the original model through perturbation theory. The reduced theory has some advantages over the original one from the computational point of view. It is super-renormalizable and yields transparent relations between the (dimensionful) continuous parameters and the lattice ones. Moreover, the number of mass scales is drastically reduced: (a) the scale T, present in four dimensions is evidently absent, (b) one may also integrate out the temporal component A_0 of the gauge field, so its mass scale gT also disappears. Thus there are two mass scales less and this reduces substantially the computer time needed to get reliable results.

The action for the U(1) gauge–Higgs model at finite temperature is:

$$S[A_\mu(\tau,\bar{x}),\varphi(\tau,\bar{x})] = \int_0^\beta d\tau \int d^3x [\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + |D_\mu\varphi|^2 + m^2\varphi^*\varphi + \lambda(\varphi^*\varphi)^2], \quad (1)$$

where $\beta = 1/T$.

If the action is expressed in terms of Fourier components, the mass terms are of the type:

$$[(2\pi nT)^2 + (\vec{k})^2]|A_\mu(n,\vec{k})|^2 \quad (2)$$

$$[(2\pi nT)^2 + (\vec{k})^2]|\varphi(n,\vec{k})|^2 \quad (3)$$

where $n = -\infty,\ldots,\infty$.

At high temperatures T and energy scales less than $2\pi T$ the non–static modes $A_\mu(n\neq 0,\vec{k})$, $\varphi(n\neq 0,\vec{k})$ are then supressed by the factor $(2\pi nT)^2$ relative to the static $A_\mu(n=0,\vec{k})$ and $\varphi(n=0,\vec{k})$ modes. The method of dimensional reduction consists in integrating out the non–static modes in the action and deriving an

effective action ([2]). We notice that the mass of the adjoint Higgs field is of order gT, which is large compared to g^2T, the typical scale of the theory. Thus, we integrate it out using perturbation theory ([3]).

The effective action may then be written in the form:

$$S_{3D\ eff}[A_i(\vec{x}), \varphi_3(\vec{x})] = \int d^3x[\frac{1}{4}F_{ij}F_{ij} + |D_i\varphi_3|^2 + m_3^2\varphi_3^*\varphi_3 + \lambda_3(\varphi_3^*\varphi_3)^2] \quad (4)$$

The index 3 in (4) is for the 3D character of the theory. The relations between 4D and 3D parameters are (up to 1–loop):

$$g_3^2 = g^2(\mu)T, \quad (5)$$

$$\lambda_3 = T\lambda(\mu) - \frac{g_3^4}{8\pi m_D}, \quad (6)$$

$$m_3^2(\mu_3) = \frac{1}{4}g_3^2T + \frac{1}{3}(\lambda_3 + \frac{g_3^4}{8\pi m_D})T - \frac{g_3^2 m_D}{4\pi} - \frac{1}{2}m_H^2$$

$$m_D^2 = \frac{1}{3}g^2(\mu)T^2. \quad (7)$$

It is convenient to use the new set of parameters (g_3^2, x, y) rather than the set $(g_3^2, \lambda_3, \mu_3^2)$. x, y are defined as ([4]):

$$x = \frac{\lambda_3}{g_3^2} \quad (8)$$

$$y = \frac{m_3^2(g_3^2)}{g_3^4} \quad (9)$$

It is evident that x is just proportional to the ratio of the squares of the scalar and vector masses; y is related to the temperature. The parameters x, y can be expressed in terms of the four–dimensional parameters as follows([5]):

$$x = \frac{1}{2}\frac{m_H^2}{m_W^2} - \frac{\sqrt{3}\ g}{8\pi} \quad (10)$$

$$y = \frac{1}{4g^2} + \frac{1}{3g^2}(x + \frac{\sqrt{3}\ g}{8\pi}) - \frac{1}{4\pi\sqrt{3}g} - \frac{m_H^2}{2g^4T^2} \quad (11)$$

We have concentrated on the phase transition line. We have chosen to fix the Higgs mass to a fixed value (30 GeV), g to 1/3, m_W to 80.6 GeV and study the characteristics of the phase transition.

2 The improved lattice action

The whole idea of improving the lattice action has been put forward ([6], [7], [8], [9], [10]) to enhance performance of the lattice calculations. The lattice actions, when expanded in powers of the lattice spacing a, yield the terms of the continuum action plus subleading terms, i.e. terms multiplied by higher powers of a. The procedure is to include additional terms in the action, so that the corrections that remain in the naïve continuum limit start at a higher power of the lattice spacing, as compared to the usual action. This work follows most closely [10].

2.1 The pure gauge part

The pure gauge part is expressed by the plaquette term in the action. We use the non-compact formulation; the initial action is defined by $\beta_g \sum_x \sum_{0<i<j} F_{ij}^2$, where $F_{ij} \equiv \Delta_i^f A_j(x) - \Delta_j^f A_i(x)$, $\Delta_i^f A_j(x) \equiv A_j(x+\hat{i}) - A_j(x)$. In the following we treat the part of the action having to do with the xy plane, i.e. we consider a two dimensional model; generalization to include the remaining hyperplanes is straightforward. In addition, we consider for both the gauge and the scalar field part of the action *two* versions of the improvement: the "continuum" and the lattice version. In the former case, a lattice with spacing equal to a is embedded in the continuum space-time and objects resembling the usual lattice quantities are considered. An interpolation is used, which makes it easy to eliminate all of the a^2 subleading terms both in the gauge and the scalar field sectors. The lattice approach is the treatment of the actual lattice model. It is not in general possible to eliminate all of the a^2 terms and one should be content with a partial cancellation.

2.1.1 Lattice embedded in the continuum

We consider the quantities:

$$C_{11} \equiv \int_{-\frac{1}{2}}^{+\frac{1}{2}} dt A_x(x+at, y-\frac{a}{2}) + \int_{-\frac{1}{2}}^{+\frac{1}{2}} dt A_y(x+\frac{a}{2}, y+at)$$

$$- \int_{-\frac{1}{2}}^{+\frac{1}{2}} dt A_x(x+at, y+\frac{a}{2}) - \int_{-\frac{1}{2}}^{+\frac{1}{2}} dt A_y(x-\frac{a}{2}, y+at),$$

$$C_{12} \equiv \int_{-1}^{+1} dt A_x(x+at, y-\frac{a}{2}) + \int_{-\frac{1}{2}}^{+\frac{1}{2}} dt A_y(x+a, y+at)$$

$$- \int_{-1}^{+1} dt A_x(x+at, y+a) - \int_{-\frac{1}{2}}^{+\frac{1}{2}} dt A_y(x-a, y+at),$$

$$C_{21} \equiv \int_{-\frac{1}{2}}^{+\frac{1}{2}} dt A_x(x+at, y-a) + \int_{-1}^{+1} dt A_y(x+\frac{a}{2}, y+at)$$

$$-\int_{-\frac{1}{2}}^{+\frac{1}{2}} dt A_x(x+at, y+a) - \int_{-1}^{+1} dt A_y(x-\frac{a}{2}, y+at).$$

Notice that C_{11} represents a continuum version of the 1×1 plaquette, while the two terms $C_{12},\ C_{21}$ represent the 1×2 and 2×1 plaquettes.

One then expands these quantities in powers of a and ends up with:

$$a^{-1}C_{11} \simeq [\partial_x A_y - \partial_y A_x] + \frac{1}{24}a^2[\partial_{xxx}A_y - \partial_{yyy}A_x + \partial_{xyy}A_y - \partial_{xxy}A_x] \quad (12)$$

$$a^{-1}C_{12} \simeq 2[\partial_x A_y - \partial_y A_x] + \frac{1}{12}a^2[4\partial_{xxx}A_y - \partial_{yyy}A_x + \partial_{xyy}A_y - 4\partial_{xxy}A_x] \quad (13)$$

$$a^{-1}C_{21} \simeq 2[\partial_x A_y - \partial_y A_x] + \frac{1}{12}a^2[\partial_{xxx}A_y - 4\partial_{yyy}A_x + 4\partial_{xyy}A_y - \partial_{xxy}A_x] \quad (14)$$

The terms that will appear in the action can be written more simply by using the notations: $F \equiv \partial_x A_y - \partial_y A_x,\ P \equiv \partial_{xxx}A_y - \partial_{yyy}A_x + \partial_{xyy}A_y - \partial_{xxy}A_x$. They read, up to $O(a^2)$:

$$a^{-2}(C_{11})^2 \simeq F^2 + \frac{1}{12}a^2 FP, \quad a^{-2}(C_{12})^2 + a^{-2}(C_{21})^2 \simeq 8F^2 + \frac{5}{3}a^2 FP. \quad (15)$$

Now it is easy to write down the expression for the action up to $O(a^2)$:

$$a^{-2}S = \sum_{x,y}[Aa^{-2}(C_{11})^2 + Ba^{-2}\{(C_{12})^2 + (C_{21})^2\}]$$

$$\simeq \sum_{x,y}[A(F^2 + \frac{1}{12}a^2 FP) + B(8F^2 + \frac{5}{3}a^2 FP)]. \quad (16)$$

We observe that in general we get the continuum action plus the FP terms, which are lattice artifacts. If our aim is to better approach the continuum action, we should arrange that these artificial terms vanish; in addition, the coefficient of the F^2 term should be one. This is easy in this approach: we just choose $A = \frac{5}{3},\quad B = -\frac{1}{12}$. Thus the improved action reads:

$$S = \sum_{x,y,\mu<\nu}[\frac{5}{3}(C_{11}^{\mu\nu}(x,y))^2 - \frac{1}{12}(C_{12}^{\mu\nu}(x,y))^2 - \frac{1}{12}(C_{21}^{\mu\nu}(x,y))^2]$$

2.1.2 Actual lattice formulation

In actual lattice calculations one cannot use the interpolation of the previous section, namely the one based on the t–integrations. One has link variables and the plaquettes used in the action are sums of four (or six) such variables. It is therefore very interesting to find out how the above analysis is modified if the real situation on the lattice is considered. We will use the same plaquettes as above and Taylor expand in powers of a. To be specific we note that the 1×1 plaquette is the sum:

$$A_x(x, y-\frac{a}{2}) + A_y(x+\frac{a}{2}, y) - A_x(x, y+\frac{a}{2}) - A_y(x-\frac{a}{2}, y) \quad (17)$$

while the 2×1 and 1×2 plaquettes are the sums:

$$\begin{aligned} & A_x(x, y-a) + A_y(x+\frac{a}{2}, y-\frac{a}{2}) + A_y(x+\frac{a}{2}, y+\frac{a}{2}) \\ & - A_x(x, y+a) - A_y(x-\frac{a}{2}, y+\frac{a}{2}) - A_y(x-\frac{a}{2}, y-\frac{a}{2}), \end{aligned} \tag{18}$$

and

$$\begin{aligned} & A_x(x-\frac{a}{2}, y-\frac{a}{2}) + A_x(x+\frac{a}{2}, y-\frac{a}{2}) + A_y(x+a, y) \\ & - A_x(x-\frac{a}{2}, y+\frac{a}{2}) - A_x(x+\frac{a}{2}, y+\frac{a}{2}) - A_y(x-a, y) \end{aligned} \tag{19}$$

respectively. The result of the expansion in a is:

$$a^{-1}P_{11} = (\partial_x A_y - \partial_y A_x) + \frac{a^2}{24}(\partial_{xxx} A_y - \partial_{yyy} A_x) + O(a^4)$$

The results for the two remaining plaquettes are:

$$a^{-1}P_{12} = 2(\partial_x A_y - \partial_y A_x) + \frac{a^2}{12}(\partial_{xxx} A_y + 3\partial_{xyy} A_y - 4\partial_{yyy} A_x) + O(a^4)$$

$$a^{-1}P_{21} = 2(\partial_x A_y - \partial_y A_x) + \frac{a^2}{12}(-\partial_{yyy} A_x - 3\partial_{xxy} A_x + 4\partial_{xxx} A_y) + O(a^4)$$

It is straightforward to verify the following:

$$a^{-2}P_{11}^2 = F^2 + \frac{a^2}{12}(-(\partial_{xx} A_y)^2 - (\partial_{yy} A_x)^2 + \partial_{xy} A_x \partial_{xx} A_y + \partial_{xy} A_y \partial_{yy} A_x) + O(a^4)$$

$$\begin{aligned} a^{-2}(P_{12}^2 + P_{21}^2) = 8F^2 - \frac{5a^2}{3}[(\partial_{xx} A_y)^2 + (\partial_{yy} A_x)^2] + O(a^4) \\ + \frac{8a^2}{3}[(\partial_{xx} A_y)(\partial_{xy} A_x) + (\partial_{yy} A_x)(\partial_{xy} A_y)] \\ - a^2[(\partial_{xy} A_x)^2 + (\partial_{xy} A_y)^2] \end{aligned} \tag{20}$$

Now we may consider the linear combination found above and see what is the outcome:

$$a^{-2}S = \frac{5}{3}a^{-2}P_{11}^2 - \frac{1}{12}a^{-2}(P_{12}^2 + P_{21}^2),$$

with the Taylor expansion:

$$F^2 + \frac{a^2}{12}[(\partial_x F)^2 + (\partial_y F)^2] \tag{21}$$

$$+ \frac{29a^2}{18}[(\partial_{xx} A_y)(\partial_{xy} A_x) + (\partial_{yy} A_x)(\partial_{xy} A_y) - (\partial_{xx} A_y)^2 - (\partial_{yy} A_x)^2] + O(a^4) \tag{22}$$

We observe that several a^2 terms do not vanish with this (or any other) choice of parameters. We have a difficulty, stemming from the nature of the actual lattice

expression of the gauge fields. A possibility to eliminate these unwanted terms might be to employ further different kinds of plaquettes; however, what really happens is that, when bigger plaquettes are considered, new terms appear that cannot vanish against existing terms.

An interesting remark is that one may choose different coefficients from the ones based on [10] and find nicer expressions on the right hand side:

$$-\frac{1}{11}P_{11}^2+\frac{3}{22}(P_{12}^2+P_{21}^2)=F^2-\frac{3a^2}{22}[(\partial_x F)^2+(\partial_y F)^2].$$

We have decided to use the standard non-compact Wilson action (with no improvement) for the gauge part, mainly because of this difficulty and the consideration of the fact that we intend to use the action in the weak gauge coupling regime, so the subleading terms are not expected to be too serious.

2.2 The gauge-scalar part

We now go ahead with the gauge-scalar sector of the action. The part that needs improvement is of course the kinetic term, the only term involving derivatives. We feel that it is important to improve this part mainly, since its big autocorrelation times make mandatory a quicker approach to the continuum limit. Following the scheme of the previous subsection, we first consider the lattice embedded in a continuous space time and afterwards we turn to the actual problem that we face on the lattice.

2.2.1 Lattice embedded in the continuum

We start by writing down the continuum kinetic term in the $\hat{\mu}$ direction for the scalar field:

$$\phi^*(\partial_\mu - iA_\mu)^2\phi + h.c. = \phi^{*\prime\prime}\phi - \phi^*\phi A^2 - i\phi^*\phi A' - 2i\phi^{*\prime}\phi A + h.c., \tag{23}$$

where the primes denote differentiations with respect to x_μ and by A we understand A_μ.

The kinetic term in the continuum involves the expression:

$$\phi^*(x)Pe^{\int_x^{x+a\hat{\mu}} A_\mu dx^\mu}\phi(x+a\hat{\mu}) + h.c. \tag{24}$$

Thus we choose to approximate this kinetic term by the expression:

$$S_{h1} \equiv \phi^*(x)e^{ia\int_0^1 dtA(x+at\hat{\mu})}\phi(x+a\hat{\mu}) + h.c. \tag{25}$$

and Taylor expand it in powers of a. The result is found to be:

$$\phi^{*\prime\prime}\phi - \phi^*\phi A^2 - i\phi^*\phi A' - 2i\phi^{*\prime}\phi A$$

$$+a^2(-\frac{1}{3}\phi^*\phi AA'' - \frac{1}{12}\phi^*\phi A^4 - \frac{1}{4}\phi^*\phi A'^2 + \frac{i}{2}\phi^*\phi A^2 A')$$

$$+a^2(-\frac{i}{3}\phi^{*\prime}\phi A'' + \frac{i}{3}\phi^{*\prime}\phi A^3 - \frac{i}{12}\phi^*\phi A''' - \frac{i}{3}\phi^{*\prime\prime\prime}\phi A - \frac{i}{2}\phi^{*\prime\prime}\phi A')$$
$$+a^2(-\frac{1}{2}\phi^{*\prime\prime}\phi A^2 - \phi^{*\prime}\phi AA' + \frac{1}{12}\phi^{*\prime\prime\prime\prime}\phi) + O(a^4) + h.c.$$

As we would like to eliminate the subleading terms, we can add next-to-nearest neighbour terms with suitable coefficients. We consider only up to second neighbours, that is we consider terms of the form:

$$S_{h2} \equiv \phi^*(x) e^{ia\int_0^2 dtA(x+at\hat{\mu})}\phi(x+2a\hat{\mu}) + h.c. \tag{26}$$

The result of the Taylor expansion contains terms qualitatively similar to the previous ones:

$$4(\phi^{*\prime\prime}\phi - \phi^*\phi A^2 - i\phi^*\phi A' - 2i\phi^{*\prime}\phi A)$$
$$+16a^2(-\frac{1}{3}\phi^*\phi AA'' - \frac{1}{12}\phi^*\phi A^4 - \frac{1}{4}\phi^*\phi A'^2 + \frac{i}{2}\phi^*\phi A^2 A')$$
$$+16a^2(-\frac{i}{3}\phi^{*\prime}\phi A'' + \frac{i}{3}\phi^{*\prime}\phi A^3 - \frac{i}{12}\phi^*\phi A''' - \frac{i}{3}\phi^{*\prime\prime\prime}\phi A - \frac{i}{2}\phi^{*\prime\prime}\phi A')$$
$$+16a^2(-\frac{1}{2}\phi^{*\prime\prime}\phi A^2 - \phi^{*\prime}\phi AA' + \frac{1}{12}\phi^{*\prime\prime\prime\prime}\phi) + O(a^4) + h.c.$$

It is easily seen that it is possible to choose the coefficients such that the a^2 subleading terms vanish. One need only consider the combination $+\frac{4}{3}S_{h1} - \frac{1}{12}S_{h2}$. It is trivial to check that the result for the Taylor expansion of this combination reads:

$$\phi^{*\prime\prime}\phi - \phi^*\phi A^2 - i\phi^*\phi A' - 2i\phi^{*\prime}\phi A + O(a^4) + h.c.,$$

which is, actually the continuum action (23).

2.2.2 Actual lattice formulation

As in the previous case, on the lattice we don't have exactly the forms (25,26) for the kinetic terms. The scalar field kinetic term before improvement reads: $\sum_{x,\hat{\mu}} \phi^*(x)U_{x\hat{\mu}}\phi(x+a\hat{\mu}) + h.c.$ To begin with, we write down the Taylor expansion of the expression $S_{h1}^{latt} \equiv \phi^*(x)e^{iaA_\mu(x+\frac{a}{2}\hat{\mu})}\phi(x+a\hat{\mu}) + h.c.$ The result is:

$$\phi^{*\prime\prime}\phi - i\phi^*\phi A' - 2i\phi^{*\prime}\phi A - \phi^*\phi A^2 + h.c.$$
$$+a^2(-\frac{1}{4}\phi^*\phi A'^2 - \frac{1}{12}\phi^*\phi A^4 + \frac{1}{3}i\phi^{*\prime}\phi A^3)$$
$$+a^2(\frac{1}{2}i\phi^*\phi A^2 A' - \frac{1}{2}\phi^{*\prime\prime}\phi A^2 - \phi^{*\prime}\phi AA')$$
$$+a^2(-\frac{1}{3}i\phi^{*\prime\prime\prime}\phi A + \frac{1}{12}\phi^{*\prime\prime\prime\prime}\phi - \frac{1}{2}i\phi^{*\prime\prime}\phi A')$$
$$+a^2(-\frac{1}{4}\phi^*\phi A'' A - \frac{1}{4}i\phi^{*\prime}\phi A'' - \frac{1}{24}i\phi^*\phi A''') + O(a^4) + h.c. \tag{27}$$

We see immediately that we have terms of order a^2, which we would like to discard by the improved action. Our step towards the improvement will be to consider the next-to-nearest neighbor terms, namely $\sum_{x,\hat{\mu}} \phi^*(x) U_{x\hat{\mu}} U_{x+\hat{\mu},\hat{\mu}} \phi(x+2a\hat{\mu}) + h.c.$

In the following lines we give the Taylor expansion of the expression $S_{h2}^{latt} \equiv \phi^*(x) e^{iaA_\mu(x+\frac{a}{2}\hat{\mu})} e^{iaA_\mu(x+\frac{3a}{2}\hat{\mu})} \phi(x+2a\hat{\mu}) + h.c.$:

$$\begin{aligned}
&4\phi^{*\prime\prime}\phi - 4i\phi^*\phi A' - 8i\phi^{*\prime}\phi A - 4\phi^*\phi A^2 \\
&+a^2(-4\phi^*\phi A'^2 - \frac{4}{3}\phi^*\phi A^4 + \frac{16}{3} i\phi^{*\prime}\phi A^3) \\
&+a^2(8i\phi^*\phi A^2 A' - 8\phi^{*\prime\prime}\phi A^2 - 16\phi^{*\prime}\phi A A') \\
&+a^2(-\frac{16}{3} i\phi^{*\prime\prime\prime}\phi A + \frac{4}{3}\phi^{*\prime\prime\prime\prime}\phi - 8i\phi^{*\prime\prime}\phi A') \\
&+ a^2(-5\phi^*\phi A'' A - 5i\phi^{*\prime}\phi A'' - \frac{7}{6} i\phi^*\phi A''') + O(a^4) + h.c.
\end{aligned} \tag{28}$$

The next natural step to be taken is, of course, to use the coefficients found before and see whether the a^2 subleading contributions go away or not. The result for the linear combination $+\frac{4}{3}S_{h1}^{latt} - \frac{1}{12}S_{h2}^{latt}$ is:

$$\phi^{*\prime\prime}\phi - i\phi^*\phi A' - 2i\phi^{*\prime}\phi A - \phi^*\phi A^2 + a^2(\frac{1}{12}\phi^*\phi A'' A + \frac{1}{12} i\phi^{*\prime}\phi A'' + \frac{1}{24} i\phi^*\phi A''') \tag{29}$$

We observe that the a^2 contributions do not vanish completely; the terms in the last lines of equations (27) and (28) survive. However, we observe that a good deal of the a^2 contributions (9 terms out of 12), present in the non-improved expression (27) have disappeared. The conclusion is that we don't really manage to get rid of all the subleading contributions of order a^2, but we expect that we approach closer to the continuum limit, so, presumably, the numerical behaviour of the improved action should be better; this has been confirmed by the simulations.

Thus, gathering everything together, we conclude that the improved action reads:

$$\begin{aligned}
S = \beta_g \sum_x \sum_{0<i<j} F_{ij}^2 + \beta_h \sum_x \sum_{0<i} [\frac{4}{3}(\varphi^*(x)\varphi(x) - \varphi^*(x) U_i(x) \varphi(x+\hat{i})) \\
-\frac{1}{12}(\varphi^*(x)\varphi(x) - \varphi^*(x) U_i(x) U_i(x+\hat{i}) \varphi(x+2\hat{i}))] \\
+ \sum_x [(1 - 2\beta_R - 3\frac{5}{4}\beta_h)\varphi^*(x)\varphi(x) + \beta_R(\varphi^*(x)\varphi(x))^2]
\end{aligned} \tag{30}$$

where $F_{ij} = \Delta_i^f A_j(x) - \Delta_j^f A_i(x)$.

The lattice parameters and the (three–dimensional) continuum ones are related as follows:

$$\beta_g = \frac{1}{a g_3^2} \tag{31}$$

$$\beta_R = \frac{x\beta_h^2}{4\beta_g} \tag{32}$$

$$2\beta_g^2 \frac{1 - 3\frac{5}{4}\beta_h - 2\beta_R}{\beta_h} = y - (1+4x)\frac{\Sigma'\beta_g}{4\pi} - \frac{\Sigma\beta_g}{4\pi} - \frac{\beta_g}{12} \tag{33}$$

where $\Sigma = 3.176$ and $\Sigma' = 2.752$. In the appendix we prove the relation (33).

3 Results

We used the Metropolis algorithm for the updating of both the gauge and the Higgs field. It is known that the scalar fields have much longer autocorrelation times than the gauge fields. Thus, special care must be taken to increase the efficiency of the updating for the Higgs field. We made the following additions to the Metropolis updating procedure [4]:

a) Global radial update: We update the radial part of the Higgs field by multiplying it by the same factor at all sites: $R(\vec{x}) \to e^{\xi} R(\vec{x})$, where $\xi \in [-\varepsilon, \varepsilon]$ is randomly chosen. The quantity ε is adjusted such that the acceptance rate is kept between 0.6 and 0.7. The probability for the updating is $P(\xi) = \min\{1, \exp(2V\xi - \Delta S(\xi))\}$ where $\Delta S(\xi)$ is the change in action, while the $2V\xi$ term comes from the change in the measure.

b) Higgs field overrelaxation: We write the Higgs potential at $\vec{x}$ in the form:

$$V(\varphi(\vec{x})) = -\mathbf{a}\cdot\mathbf{F} + R^2(\vec{x}) + \beta_R(R^2(\vec{x}) - 1)^2 \tag{34}$$

where

$$\mathbf{a} \equiv \begin{pmatrix} R(\vec{x})\cos\chi(\vec{x}) \\ R(\vec{x})\sin\chi(\vec{x}) \end{pmatrix},$$

$$\mathbf{F} \equiv \begin{pmatrix} F_1 \\ F_2 \end{pmatrix},$$

$$F_1 \equiv \beta_h \sum_i [\frac{4}{3} R(\vec{x}+\hat{i})\cos(\chi(\vec{x}+\hat{i})+\theta(\vec{x})) - \frac{1}{12} R(\vec{x}+2\hat{i})\cos(\chi(\vec{x}+2\hat{i})+\theta(\vec{x})+\theta(\vec{x}+\hat{i}))],$$

$$F_2 \equiv \beta_h \sum_i [\frac{4}{3} R(\vec{x}+\hat{i})\sin(\chi(\vec{x}+\hat{i})+\theta(\vec{x})) - \frac{1}{12} R(\vec{x}+2\hat{i})\sin(\chi(\vec{x}+2\hat{i})+\theta(\vec{x})+\theta(\vec{x}+\hat{i}))].$$

We can perform the change of variables: $(\mathbf{a}, \mathbf{F}) \to (X, F, \mathbf{Y})$,where

$$F \equiv |\mathbf{F}|, \quad \mathbf{f} \equiv \frac{\mathbf{F}}{\sqrt{F_1^2 + F_2^2}}, \quad X \equiv \mathbf{a}\cdot\mathbf{f}, \quad \mathbf{Y} \equiv \mathbf{a} - X\mathbf{f}. \tag{35}$$

The potential may be rewritten in terms of the new variables:

$$\bar{V}(X, F, \mathbf{Y}) = -XF + (1 + 2\beta_R(\mathbf{Y}^2 - 1))X^2 + \mathbf{Y}^2(1 - 2\beta_R) + \beta_R(X^4 + \mathbf{Y}^4). \tag{36}$$

The updating of $\mathbf{Y}$ is done simply by the reflection:

$$\mathbf{Y} \to \mathbf{Y}' = -\mathbf{Y}. \tag{37}$$

The updating of X is performed by solving the equation:

$$\bar{V}(X', F, \mathbf{Y}) = \bar{V}(X, F, \mathbf{Y}) \tag{38}$$

with respect to X'. Noting that $X' = X$ is obviously a solution, we may factor out the quantity $X' - X$ and reduce the quartic equation into a cubic one, which may be solved. The change $X \to X'$ is accepted with probability: $P(X') = \min\{P_0, 1\}$, where $P_0 \equiv \frac{\partial \bar{V}(X,F,\mathbf{Y})}{\partial X} / \frac{\partial \bar{V}(X',F',\mathbf{Y}')}{\partial X'}$.

For our Monte–Carlo simulations we used cubic lattices with volumes $V = 8^3, 12^3, 16^3$. For each volume we performed 30000–50000 thermalization sweeps and 60000–100000 measurements. We have set the value of x equal to 0.0463. According to the relation (10) using $m_W = 80.6 GeV$ and $g = \frac{1}{3}$ this value of x corresponds to Higgs field mass $m_H = 30 GeV$. For each value of β_h we determine the value of β_R by using relation (32). The phase transition is expected to be of first order for such a low mass of the scalar field.

We have used five quantities to determine the phase transition points:

1. The distribution $N(E_{link})$ of E_{link}.

2. The susceptibility of $E_{link} \equiv \frac{1}{3V} \sum_{x,i} \Omega^*(x) U_i(x) \Omega(x+i)$ (we have set $\varphi(x) \equiv R(x) e^{i\chi(x)} \equiv R(x)\Omega(x)$):

$$S(E_{link}) \equiv V(< (E_{link})^2 > - < E_{link} >^2).$$

3. The susceptibility of $R2 \equiv \frac{1}{V} \sum_x R^2(x)$:

$$S(R2) \equiv V(< (R2)^2 > - < R2 >^2).$$

4. The Binder cumulant of E_{link}:

$$C(E_{link}) = 1 - \frac{< (E_{link})^4 >}{3 < (E_{link})^2 >^2}.$$

5. The Binder cumulant of $R \equiv \frac{1}{V} \sum_x R(x)$:

$$C(R) = 1 - \frac{< (R)^4 >}{3 < (R)^2 >^2}.$$

The pseudocritical $\beta_h^*(A, V)$ values have been found by determining (a) equal heights of the two peaks of the distribution $N(E_{link})$, (b) the maxima of the quantities $S(E_{link})$, $S(R2)$ and (c) the minima of the cumulants $C(E_{link}), C(R)$. The values $\beta_h^*(A, V)$ depend on the specific quantity (denoted by A) which has

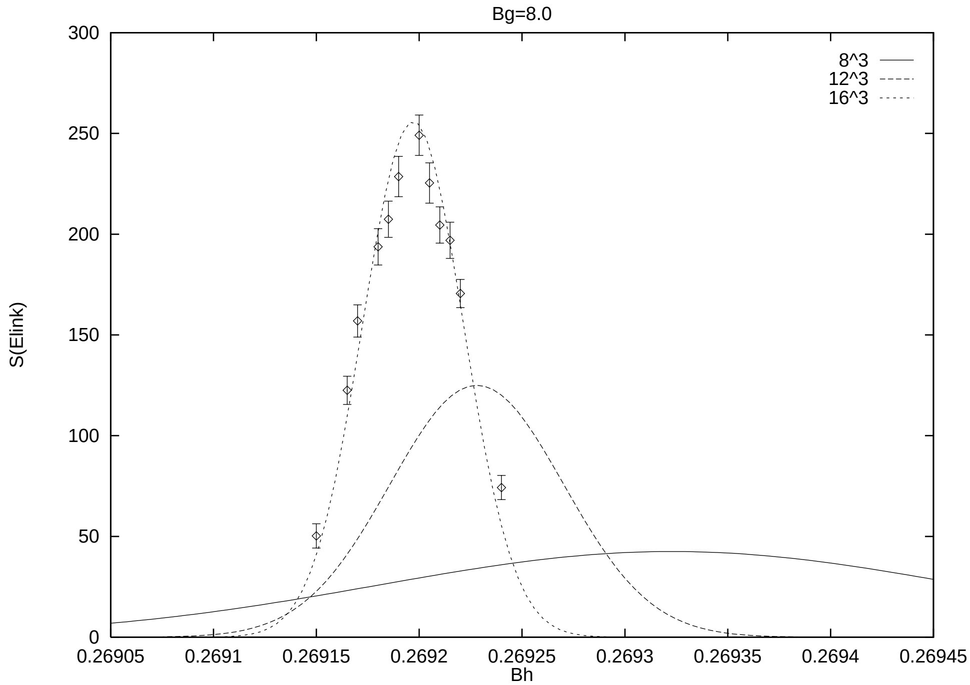

Figure 1: Susceptibility of E_{link}

been employed, as well as on the volume V. It has to be noticed that while searching we have made use of the Ferrenberg–Swendsen reweighting technique ([12]) to find the pseudocritical β_h for the volume 16^3.

In figure 1 we depict the behaviour of the susceptibility $S(E_{link})$ versus β_h for three lattice volumes. The curves are fitted through the data; for simplicity we give the actual measurements for the largest volume. The curves represent the data quite nicely. In calculating the error bars we first found the integrated auto-correlation times $\tau_{int}(A)$ for the relevant quantities A and constructed samples of data separated by a number of steps greater than $\tau_{int}(A)$. Then the errors have been calculated by the Jacknife method ([13]), using the samples constructed according to the procedure just described. Notice that the peak values increase almost linearly with the volume which is characteristic of a first order transition.

In figures 2 and 3 we depict the behaviour of the Binder cumulants $C(E_{link})$ and $C(R)$ for three lattice sizes. Again, we show the real measurements for the largest volume. The error bars have been calculated, also, by the Jacknife method. The volume dependence of the cumulants display evidence for a first order phase transition.

The different values of $\beta_h^*(A,V)$ corresponding to the quantities A are due to the finite lattices used. So, we should extrapolate these values to infinite volume adopting the ansatz:

$$\beta_h^*(A,\ V) = \beta_h^{cr}(\infty) + \frac{c(A)}{V},$$

The extrapolated value $\beta_h^{cr}(\infty)$ should not depend on quantity A because this

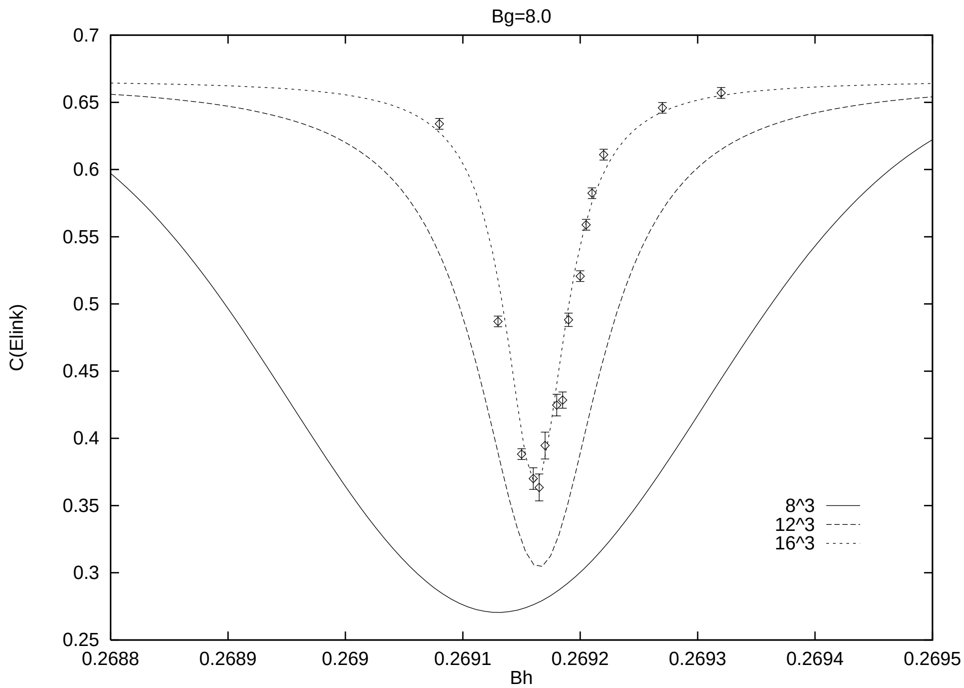

Figure 2: Cumulant of E_{link}

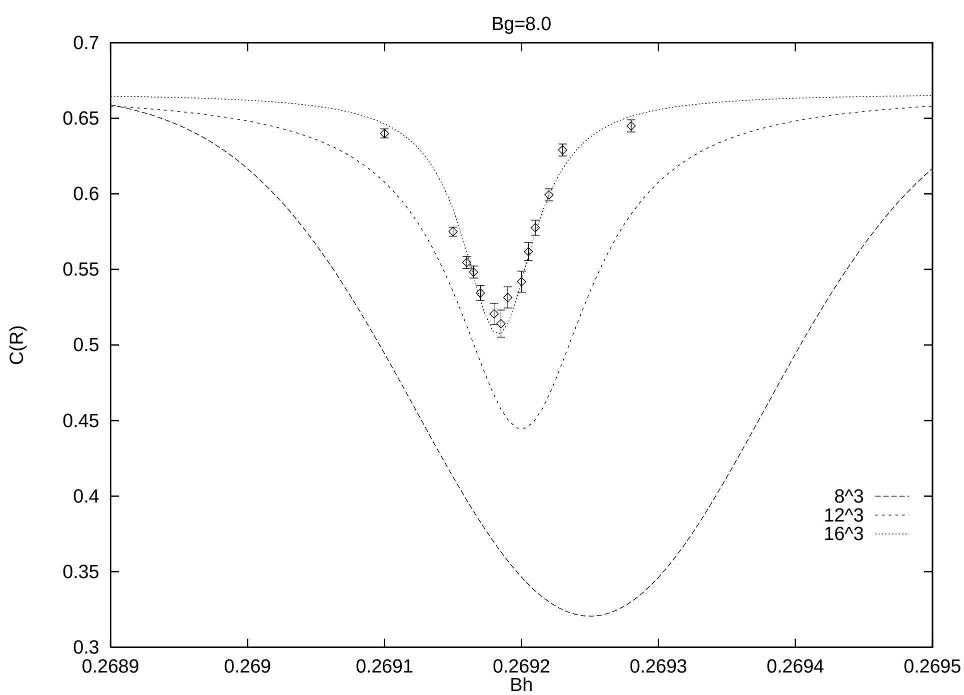

Figure 3: Cumulant of R

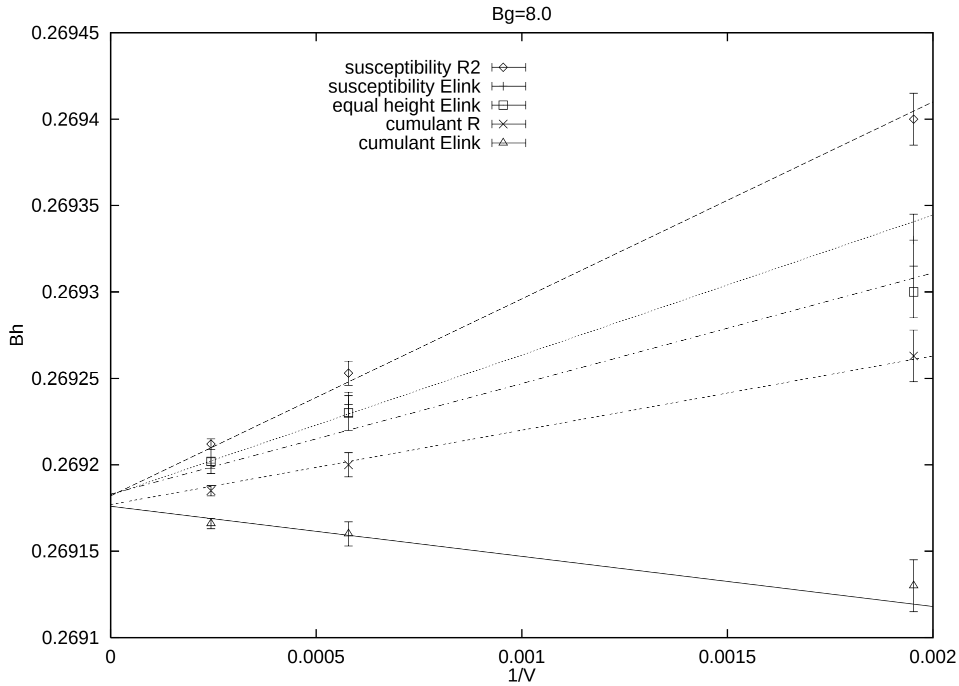

Figure 4: Extrapolation for $\beta_g = 8$

is the infinite volume extrapolation for the critical point.

Figure 4 shows the extrapolation to infinite volume using data for the pseudo-critical $\beta_h^*(A,V)$ values obtained from the various quantities A versus the inverse lattice volume, along with the linear fits to the data. The error bars in $\beta_h^*(A,V)$ have been calculated from the statistical error of the values of the quantities A at the critical point. One can observe that, at, finite volumes, the smallest pseudocritical values are given by the cumulant of E_{link}; then, in ascending order, the values given by the cumulant of R, the equal height, the susceptibility of E_{link} and the susceptibility of $R2$. Also, we notice that the infinite volume extrapolation is almost independent from the specific quantity used; the differences at the point $\frac{1}{V} = 0$ between the various extrapolated values are less than 10^{-5}. The critical values lie in the interval (0.269176, 0.269183). In our previous publication [1] where we worked with the same model, but without any improvement to the Higgs part of the action, we had found that for the same value of β_g the critical β_h values were lying in the interval (0.336932, 0.336940). Although the precision is comparable, one should notice that the result presented here has been found by using three times smaller lattice volumes than the previous one which means much shorter computer time. Evidently, this result is due to the effect of improving the lattice Higgs action which provides a quicker approach to the thermodynamic limit.

We can, now, predict the critical temperature T_{cr}. Actually, the quantity β_h^{cr} yields y_{cr} through equations (32, 33); then equation (11) gives $T_{cr} = 130.64(19)$. In [1] using 2–loop calculations it was found $T_{cr} = 131.18(14)$ while a 1–loop

calculation would give $T_{cr} = 130.74(14)$. Thus, the two 1–loop results are almost identical, but the first one has been achieved in a more economical way due to the improved action used.

Acknowledgements

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4 Appendix

We want to prove the relation (33), which relates the masses on the lattice and in the continuum up to 1–loop perturbation theory. In general we have:

$$m_L^2 = m_3^2 - \delta m_L^2(h) \tag{39}$$

The counterterm will be calculated by considering the $\frac{1}{a}$ terms from the 1–loop lattice effective potential ([14]).

We consider the pure gauge and gauge fixing part of the action (30). We use the relation $\beta_g = \frac{1}{g_3^2 a}$ and write down the kinetic term as:

$$S_g = a^3 \sum_x \frac{1}{4} \sum_{i \neq j} \left(\frac{\Delta_i^f A_j - \Delta_j^f A_i}{a} \right)^2$$

Let us comment here a little bit about the gauge fixing term, which is the new element here. For the perturbative treatment of Higgs models it is usual to employ the R_ξ gauges. This choice is dictated by its simplicity, since then the mixing terms between the gauge field and the would-be Goldstone bosons vanish. However, one should be careful with this gauge, since the so-called Nielsen identities [16] should be satisfied [17]. This severely restricts the possible R_ξ gauges; violation of these identities will lead to unphysical results. This is the reason that we have chosen to stick to the Lorentz gauge fixing for the gauge action:

$$L_{gf} = \frac{a}{2\xi} \sum_x \sum_i [A_i(x) - A_i(x - \hat{\imath})]^2.$$

This gauge does not face the previous complications, so we prefer to stay on the safe side, at the price, of course, of having also to deal with the non-vanishing mixed terms. Note that we take the Landau gauge $\xi = 0$ at the end of the calculations.

Going to the Fourier space (notice that $A_\mu(x) = \int \frac{d^3p}{(2\pi)^3} e^{ip(x+\frac{a\hat{\mu}}{2})} \tilde{A}_\mu(p)$), we find:

$$S_g + S_{gf} = \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \sum_{i,j>0} \tilde{A}_i(p)[(\tilde{p}^2 + \frac{1}{a^2}(\frac{1}{\xi} - 1)\tilde{p}_i \tilde{p}_j] \tilde{A}_j(-p) \tag{40}$$

where

$$\tilde{p}_i = \frac{2}{a} sin \frac{p_i a}{2} \tag{41}$$

Next we consider the part of the action involving the scalar fields. We follow essentially the same procedure as above, but a step that should be taken is the decomposition $\phi = \frac{1}{\sqrt{2}}(\phi_0 + \phi_1 + i\phi_2)$ of the scalar field. Then the rescaling $(\frac{\beta_h}{2a})^{\frac{1}{2}} \varphi = \phi$ has to be performed to get the corresponding part of the action in continuum form. The Fourier transform of the relevant part of the action reads:

$$S_H = +\frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \phi_1(p)(\tilde{p}^2 + \frac{a^2}{12}\tilde{p}^4 + m_1^2)\phi_1(-p)$$

$$+\frac{1}{2}\int\frac{d^3p}{(2\pi)^3}\phi_2(p)(\tilde{p}^2+\frac{a^2}{12}\tilde{p}^4+m_2^2)\phi_2(-p)$$

$$+\int\frac{d^3p}{(2\pi)^3}\tilde{A}_i(p)(\frac{m_T}{\alpha}\tilde{g}_i)\phi_2(-p)+\frac{1}{2}\int\frac{d^3p}{(2\pi)^3}\tilde{A}_i(p)(m_T^2+\frac{\alpha^2 m_T^2}{12}\tilde{p}_i^2)\tilde{A}_i(-p) \quad (42)$$

with $\tilde{g}_i \equiv \frac{5}{4}\tilde{p}_i - \frac{1}{12}\hat{p}_i$. Let us collect some additional notations that have just been used:

$$\hat{p}_i \equiv \frac{2}{a}sin\frac{3p_i a}{2},\ \tilde{p}^2 \equiv \sum_i \tilde{p}_i^2,\ \tilde{p}^4 \equiv \sum_i \tilde{p}_i^4, \quad (43)$$

$$m_T^2 \equiv g_3^2\phi_0^2,\ m_1^2 \equiv m_3^2(\mu)+3\lambda_3\phi_0^2,\ m_2^2 \equiv m_3^2(\mu)+\lambda_3\phi_0^2. \quad (44)$$

Now, considering the quadratic part of the action, which is contained in the above equations, it is easy to read out the propagators D_1, D_2 of the fields ϕ_1, ϕ_2 respectively:

$$D_1^{-1}=\tilde{p}^2+\frac{a^2}{12}\tilde{p}^4+m_1^2,\quad D_2^{-1}=\tilde{p}^2+\frac{a^2}{12}\tilde{p}^4+m_2^2. \quad (45)$$

In the equation above the terms proportional to $\tilde{p}^4$ arise directly from the improvement terms in the scalar sector concerning the next-to-nearest-neighbour contribution.

The effective potential at one loop is found using the relation:

$$V_L^{1-loop}=-ln(Z),$$

where $Z \equiv \int[D\phi_1][D\phi_2][DA]e^{-S}$. Keeping only the quadratic part of the action we have just Gaussian integrations, so we easily get the result:

$$V_L^{1-loop}=\frac{1}{2}\int\frac{d^3p}{(2\pi)^3}ln(D_1^{-1})+\frac{1}{2}\int\frac{d^3p}{(2\pi)^3}ln(D_2^{-1})+$$

$$\frac{1}{2}\int\frac{d^3p}{(2\pi)^3}ln(det(\Delta_{ij}^{-1}-N_{ij})) \quad (46)$$

where

$$\Delta_{ij}^{-1}-N_{ij}=(\tilde{p}^2+m_T^2+\frac{a^2m_T^2}{12}\tilde{p}_i\tilde{p}_j)\delta_{ij}+\frac{1}{a^2}(\xi^{-1}-1)\tilde{p}_i\tilde{p}_j-\frac{m_T^2}{a^2}\frac{\tilde{g}_i\tilde{g}_j}{D_2^{-1}}, \quad (47)$$

We write the gauge propagator in this form to display the contribution $N_{ij} \equiv \frac{m_T^2}{a^2}\frac{\tilde{g}_i\tilde{g}_j}{D_2^{-1}}$, which is due to the mixing term between A_i and the imaginary part of the Higgs field.

An integral which appears very frequently and should be calculated is the following:

$$I(m)=\frac{1}{2}\int_{-\pi/a}^{\pi/a}\frac{d^3p}{(2\pi)^3}ln(\tilde{p}^2+\frac{a^2}{12}\tilde{p}^4+m^2) \quad (48)$$

If we differentiate the above with respect of m we take:

$$dI(m) = mK(m)dm \tag{49}$$

where

$$aK(m) = \int_{-\pi}^{\pi} \frac{d^3p}{(2\pi)^3} \frac{1}{\bar{p}^2 + \frac{1}{12}\bar{p}^4 + M^2} \tag{50}$$

with: $M^2 = (am)^2$, $\bar{p}^2 = 4\sum_i sin^2\frac{p_i}{2}$ and $\bar{p}^4 = 16\sum_i sin^4\frac{p_i}{2}$ At this point we follow ([15]) where the expansion of (50) in powers of $M = am$ is to be used. In the following we denote by B the "Brillouin zone" $[-\pi, +\pi]^3$, in addition, $\bar{\Pi}^2 \equiv \bar{p}^2 + \frac{1}{12}\bar{p}^4$. Then the following equality holds:

$$\int_B \frac{d^3p}{(2\pi)^3} \frac{1}{\bar{p}^2 + \frac{1}{12}\bar{p}^4 + M^2} = \int_B \frac{d^3p}{(2\pi)^3} \frac{1}{(\bar{\Pi}^2)^2} - \int_B \frac{d^3p}{(2\pi)^3} \frac{M^2}{\bar{\Pi}^2(\bar{\Pi}^2 + M^2)}$$

The first integral equals $\frac{\Sigma'}{4\pi}$; the contribution of the second integral can be shown to be of $O(a)$, so it will not contribute to the infinite part. Gathering everything together yields:

$$K(m) = \frac{\Sigma'}{4\pi a} + \text{finite}$$

Hence, integrating (49) we get:

$$I(m) = \frac{1}{2}\frac{\Sigma'}{4\pi a}m^2 + \text{finite} \tag{51}$$

where

$$\Sigma' = \frac{1}{(\pi)^2}\int_0^{\pi} d^3p \frac{1}{\sum_i (sin^2\frac{p_i}{2} + \frac{1}{3}sin^4\frac{p_i}{2}}) \tag{52}$$

is being calculated numerically at the value of $\Sigma' = 2.752$.

Up to this point we are ready to calculate the infinite part in the effective potential (behaving like $\frac{1}{a}$) that is due to the scalar fields only. The mass terms coming from the fields ϕ_1, ϕ_2 are:

$$\frac{1}{2}\frac{\Sigma'}{4\pi a}m_1^2, \quad \frac{1}{2}\frac{\Sigma'}{4\pi a}m_2^2$$

respectively. Recalling (44), their derivatives with respect to the classical field ϕ_0^2 are:

$$(3\lambda_3 + \lambda_3)\frac{1}{2}\frac{\Sigma'}{4\pi a} = 4xg_3^2\frac{1}{2}\frac{\Sigma'}{4\pi a} \tag{53}$$

The next step is to calculate the gauge contribution to the effective potential, that is the third term in equation (46). The calculations are straightforward, but quite tedious; we just mention that we directly expand the determinant and keep contributions only up to order a^2, since the a^4 terms will yield merely finite results, which are not our concern here. We only write down the final result:

$$V_L^{1-loop} = \int \frac{d^3p}{(2\pi)^3}[ln(\tilde{p}^2 + m_T^2)^2 + (ln\tilde{p}^2 - ln\xi)$$

$$+\, ln(1+2\frac{\frac{a^2m_T^2}{12}}{\tilde{p}^2+m_T^2}\frac{\tilde{p}_1^2\tilde{p}_2^2+\tilde{p}_2^2\tilde{p}_3^2+\tilde{p}_3^2\tilde{p}_1^2}{\tilde{p}^2})+O(a^4)] \tag{54}$$

We note that the second term is exactly the same as the one appearing in the continuum counterpart of the model, so they cancel upon comparison with the continuum model. These terms do not depend on the mass, so they present no interest for the calculation of the effective potential anyway. We should note that we have not given the full result of the third term; to keep things simple, we gave its value for $\xi = 0$. The above expression can be written as:

$$\frac{1}{2}\int_{-\pi/a}^{\pi/a}\frac{d^3p}{(2\pi)^3}ln(\tilde{p}^2+\frac{a^2}{12}\tilde{p}^4+m_T^2)+\frac{1}{2}\int_{-\pi/a}^{\pi/a}\frac{d^3p}{(2\pi)^3}ln(\tilde{p}^2+m_T^2)+\frac{1}{2}\frac{1}{12a}m_T^2 \tag{55}$$

The integrals appearing in equation (55) have already been computed; the infinite part of the effective potential reads:

$$\frac{1}{2}(\frac{\Sigma'}{4\pi a}m_T^2+\frac{\Sigma}{4\pi a}m_T^2+\frac{1}{12a}m_T^2) \tag{56}$$

We invoke equation (44) and calculate again the second derivative with respect to the classical field ϕ_0 to find out that the infinite part reads:

$$\frac{g_3^2}{2}(\frac{\Sigma'}{4\pi a}+\frac{\Sigma}{4\pi a}+\frac{1}{12a}) \tag{57}$$

Collecting all the infinite contributions we can match the counterterm equation and this yields:

$$2\beta_g^2(\frac{1}{\beta_h}-3\frac{5}{4}-\frac{2\beta_R}{\beta_h})=y-(1+4x)\frac{\Sigma'\beta_g}{4\pi}-\frac{\Sigma\beta_g}{4\pi}-\frac{\beta_g}{12},$$

which is the equation (33).

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